Q1	$\mathrm{f}(x) = k(1-x) \qquad 0 \le x \le 1$			
(i)	$\int_0^1 k(1-x)\mathrm{d}x = 1$ $\therefore k[x - \frac{1}{2}x^2]_0^1 = 1$ $\therefore k(1-\frac{1}{2}) - 0 = 1$ $\therefore k = 2$ Labelled sketch: straight line segment from (0,2) to (1,0).	M1 E1 G1 G1	Integral of f(x), including limits (possibly implied later), equated to 1. Convincingly shown. Beware printed answer. Correct shape. Intercepts labelled.	 4
(ii)	$\mathrm{E}(X) = \int_0^1 2x(1-x)\mathrm{d}x$ $= [x^2 - \frac{2}{3}x^3]_0^1 = (1-\frac{2}{3}) - 0 = \frac{1}{3}$ $\mathrm{E}(X^2) = \int_0^1 2x^2(1-x)\mathrm{d}x$ $= [\frac{2}{3}x^3 - \frac{2}{4}x^4]_0^1 = (\frac{2}{3}-\frac{1}{2}) - 0 = \frac{1}{6}$ $\mathrm{Var}(X) = \frac{1}{6} - (\frac{1}{3})^2$ $= \frac{1}{18}$	M1 A1 M1 M1 A1	Integral for E(X) including limits (which may appear later). Integral for E(X^2) including limits (which may appear later). Convincingly shown. Beware printed answer.	 5
(iii)	$\mathrm{F}(x) = \int_0^x 2(1-t)\mathrm{d}t$ $= [2t - t^2]_0^x = (2x - x^2) - 0 = 2x - x^2$ $\mathrm{P}(X > \mu) = \mathrm{P}(X > \frac{1}{3}) = 1 - \mathrm{F}(\frac{1}{3})$ $= 1 - (2\times\frac{1}{3} - (\frac{1}{3})^2) = 1 - \frac{5}{9} = \frac{4}{9}$	M1 A1 M1 A1	Definition of cdf, including limits, possibly implied later. Some valid method must be seen. [for $0 \le x \le 1$; do not insist on this.] For 1 – c's F(μ). ft c's E(X) and F(x). If answer only seen in decimal expect 3 d.p. or better.	 4
(iv)	$\mathrm{F}\left(1-\frac{1}{\sqrt{2}}\right) = 2\left(1-\frac{1}{\sqrt{2}}\right) - \left(1-\frac{1}{\sqrt{2}}\right)^2$ $= 2 - \frac{2}{\sqrt{2}} - 1 + \frac{2}{\sqrt{2}} - \frac{1}{2} = \frac{1}{2}$ **Alternatively:** $2m - m^2 = \frac{1}{2}$ $\therefore m^2 - 2m + \frac{1}{2} = 0$ $\therefore m = 1 \pm \frac{1}{\sqrt{2}}$ so $m = 1 - \frac{1}{\sqrt{2}}$	M1 E1 M1 E1	Substitute $m = 1 - \frac{1}{\sqrt{2}}$ in c's cdf. Convincingly shown. Beware printed answer. Form a quadratic equation $\mathrm{F}(m) = \frac{1}{2}$ and attempt to solve it. ft c's cdf provided it leads to a quadratic. Convincingly shown. Beware printed answer.	 2
(v)	$\overline{X} \sim \mathrm{N}(\frac{1}{3}, \frac{1}{1800})$	B1 B1 B1	Normal distribution. Mean. ft c's E(X). Correct variance.	 3
				18

Q2				
(i)	$H_0 : \mu = 0{\cdot}6$ $H_1 : \mu < 0{\cdot}6$ Where μ is the (population) mean height of the saplings.	B1 B1 B1	Allow absence of "population" if correct notation μ is used, but do NOT allow "$\bar{X}=...$" or similar unless $\bar{X}$ is clearly and explicitly stated to be a <u>population</u> mean. Hypotheses in words only must include "population".	
	$\bar{x} = 0{\cdot}5883$, $s_{n-1} = 0{\cdot}03664$ ($s_{n-1}^2 = 0{\cdot}00134$)	B1	Do not allow $s_n = 0{\cdot}03507$ ($s_n^2 = 0{\cdot}00123$).	
	Test statistic is $\dfrac{0\cdot5883-0\cdot6}{\left(\dfrac{0\cdot03664}{\sqrt{12}}\right)}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: $0{\cdot}6 \pm$ (c's $-1{\cdot}796$) $\times \frac{0\cdot03664}{\sqrt{12}}$ (=0·5810, 0·6190) for subsequent comparison with $\bar{x}$. (Or $\bar{x} \pm$ (c's $-1{\cdot}796$) $\times \frac{0\cdot03664}{\sqrt{12}}$ (=0·5693, 0·6073) for comparison with 0·6.)	
	$= -1{\cdot}103$	A1	c.a.o. but ft from here in any case if wrong. Use of $0{\cdot}6 - \bar{x}$ scores M1A0, but ft.	
	Refer to t_{11}. Lower 5% point is $-1{\cdot}796$.	M1 A1	No ft from here if wrong. No ft from here if wrong. Must be $-1{\cdot}796$ unless it is clear that absolute values are being used.	
	$-1{\cdot}103 > -1{\cdot}796$, $\therefore$ Result is not significant.	E1	ft only c's test statistic.	
	Seems mean height of saplings meets the manager's requirements.	E1	ft only c's test statistic.	11
	Underlying population is Normal.	B1		
(ii)	CI is given by $0{\cdot}5883 \pm$ $2{\cdot}201$ $\times \dfrac{0\cdot03664}{\sqrt{12}}$	M1 B1 M1	ft c's $\bar{x} \pm$. ft c's s_{n-1}.	
	$= 0{\cdot}5883 \pm 0{\cdot}0233 = (0{\cdot}565(0), 0{\cdot}611(6))$	A1	c.a.o. Must be expressed as an interval. ZERO if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_{11} is OK.	

	In repeated sampling, 95% of intervals constructed in this way will contain the true population mean.	E1		5
(iii)	Could use the Wilcoxon test. Null hypothesis is "Median = 0.6".	E1 E1		2
				18

Q3	$M \sim \mathrm{N}(44,\ 4{\cdot}8^2)$ $H \sim \mathrm{N}(32,\ 2{\cdot}6^2)$ $P \sim \mathrm{N}(21,\ 3{\cdot}7^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables, penalise the first occurrence only.	
(i)	$\mathrm{P}(M < 50) = \mathrm{P}(Z < \frac{50-44}{4{\cdot}8} = 1{\cdot}25)$ $= 0{\cdot}8944$	M1 A1 A1	For standardising. Award once, here or elsewhere.	3
(ii)	$H + P \sim \mathrm{N}(32 + 21 = 53,$ $2{\cdot}6^2 + 3{\cdot}7^2 = 20{\cdot}45)$ $\mathrm{P}(H + P < 50) = \mathrm{P}(Z < \frac{50-53}{\sqrt{20{\cdot}45}} = -0{\cdot}6634)$ $= 1 - 0{\cdot}7465 = 0{\cdot}2535$	B1 B1 A1	Mean. Variance. Accept sd = $\sqrt{20{\cdot}45}$ = 4·522... c.a.o.	3
(iii)	Want $\mathrm{P}(M > H + P)$ i.e. $\mathrm{P}(M - (H + P) > 0)$ $M - (H + P) \sim \mathrm{N}(44 - (32 + 21) = -9,$ $4{\cdot}8^2 + 2{\cdot}6^2 + 3{\cdot}7^2 = 43{\cdot}49)$ $\mathrm{P}(\text{this} > 0) = \mathrm{P}(Z > \frac{0-(-9)}{\sqrt{43{\cdot}49}} = 1{\cdot}365)$ $= 1 - 0{\cdot}9139 = 0{\cdot}0861$	M1 B1 B1 A1	Allow $H + P - M$ provided subsequent work is consistent. Mean. Variance. Accept sd = $\sqrt{43{\cdot}49}$ = 6·594... c.a.o.	4
(iv)	Mean = 44 + 44 + 32 + 32 + 21 + 21 = 194 Variance = $4{\cdot}8^2 + 4{\cdot}8^2 + 2{\cdot}6^2 + 2{\cdot}6^2 + 3{\cdot}7^2 + 3{\cdot}7^2$ = 86·98	B1 B1	(sd = 9·3263…)	2
(v)	$C \sim \mathrm{N}(194 \times 0{\cdot}15 + 10 = 39{\cdot}10,$ $86{\cdot}98 \times 0{\cdot}15^2 = 1{\cdot}957)$ $\mathrm{P}(C \le 40) = \mathrm{P}(Z \le \frac{40-39{\cdot}10}{\sqrt{1{\cdot}957}} = 0{\cdot}6433)$ $= 0{\cdot}7400$ **Alternatively:** $\mathrm{P}(C \le 40) = \mathrm{P}(\text{total time} \le \frac{40-10}{0.15} = 200$ minutes) $= \mathrm{P}(Z \le \frac{200-194}{\sqrt{86{\cdot}98}} = 0{\cdot}6433)$	M1 M1 A1 M1 A1 A1 M1 M1 A1 M1 A1	c's mean in (iv) × 0·15 + 10 (or subtract 10 from 40 below) ft c's mean in (iv). c's variance in (iv) × $0{\cdot}15^2$ ft c's variance in (iv). c.a.o. – 10 ÷ 0.15 c.a.o. Correct use of c's variance in (iv). ft c's mean and variance in (iv).	6

	= 0·7400	A1	c.a.o.	
				18

Q4				
(a)	Obs 10, Exp 6·68	M1	Combine first two rows.	
	$\therefore X^2 = \dfrac{(10-6\cdot 68)^2}{6\cdot 68} + \text{etc}$	M1		
	$= 1{\cdot}6501 + 1{\cdot}7740 + 3{\cdot}3203 + 4{\cdot}5018 + 0{\cdot}4015 + 0{\cdot}8135$			
	$= 12{\cdot}46(12)$	A1		
	d.o.f. = 6 – 3 = 3		Require d.o.f. = No. cells used – 3.	
	Refer to χ_3^2.	M1	No ft from here if wrong.	
	Upper 5% point is 7·815	A1	No ft from here if wrong.	
	12·46 > 7·815 $\therefore$ Result is significant.	E1	ft only c's test statistic.	
	Seems the Normal model does not fit the data at the 5% level.	E1	ft only c's test statistic.	
	E.g. • The biggest discrepancy is in the class $1{\cdot}01 < a \le 1{\cdot}02$	E1		
	• The model overestimates in classes …, but underestimates in classes …	E1	Any two suitable comments.	9
(b)				

Old – New:	0·007	0·002	–0·001	–0·003	0·004	–0·008	–0·010	0·009	–0·005	–0·016
Rank of |diff|	6	2	1	3	4	7	9	8	5	10

		M1	For differences. ZERO in this section if differences not used.	
		M1	For ranks of |difference|.	
		A1	All correct. ft from here if ranks wrong.	
	$W_+ = 6 + 2 + 4 + 8 = 20$	B1	Or $W_- = 1 + 3 + 7 + 9 + 5 + 10 = 35$	
	Refer to Wilcoxon single sample (/paired) tables for $n = 10$.	M1	No ft from here if wrong.	
	Lower two-tail 10% point is …	M1	Or, if 35 used, upper point is 45.	
	… 10.	A1	No ft from here if wrong.	
	20 > 10 $\therefore$ Result is not significant.	E1	Or 35 < 45. ft only c's test statistic.	
	Seems there is no reason to suppose the barometers differ.	E1	ft only c's test statistic.	9
				18